Optimizing flotation bank performance by recovery profiling

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ABSTRACT

This paper uses the first-order fully mixed model to argue that operating a bank of cells with a flat cell-by-cell recovery profile yields maximum separation between two floatable minerals with constant relative floatability for a target bank cumulative recovery. The bank optimization problem thus translates into a local problem of selecting cell manipulated variables, such as air rate, to reach that recovery profile. Some properties of the bank that emerge from the analysis are discussed. Recovery profiling appears to contribute to the success of air profiling recently reported.

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1. Introduction

Flotation circuits are arrangements of stages which typically comprise several cells in series referred to as a line, row or bank. The series arrangement reduces short circuiting and provides for transport to approach plug flow and thus minimizes retention time for a target recovery (Gupta and Yan, 2006). There is, however, no clear guidance how to operate a bank to maximize mineral separation.

Xstrata Brunswick Division tested various strategies of distributing (profiling) air to the final (fourth stage) Zn cleaner bank of seven cells. It was found that an increasing profile gave the best performance (down-the-bank grade-recovery relationship) (Cooper et al., 2004). The increasing air rate profile was adopted on all four cleaner stages with total bank air adjusted to achieve target bank recovery; and it remains the practice. Other operations subsequently demonstrated that air profiling can generate significant performance gains (Gorain, 2005; Hernandez-Aguilar and Reddick, 2007; Smith et al., 2008; Hernandez-Aguilar, 2010; Hadler et al., 2010).

Analysis of the Brunswick case concluded that the improvement was due to reduced entrainment of non-sulphide (non-floatable) gangue in the first cells in the bank because the low air rate restricted water recovery. There was no difference in relative floatability of the two floatable minerals, sphalerite and pyrite, and thus it appeared there was no impact on their separation. Air profiling, however, could be considered as recovery profiling, the different air profiles distributing material differently down the bank, which opens the question addressed in this paper: does the recovery profile influence separation of floatable minerals? We address the question as an optimization problem.

2. Optimization of a bank performance to separate two floatable minerals

Two floatable minerals A and B are considered. The optimization problem is formulated to maximize the (technical) separation efficiency (Schulz, 1970), i.e., cumulative recovery of mineral A minus cumulative recovery of mineral B, for a target cumulative recovery of mineral A.

Making the common assumption of first-order flotation kinetics and fully mixed isolated cell, the recovery of mineral A and B in cell \( j \) can be expressed as follows:

\[
R_{A_j} = \frac{k_{A_j} \cdot \tau}{1 + k_{A_j} \cdot \tau}
\]

\[
R_{B_j} = \frac{k_{B_j} \cdot \tau}{1 + k_{B_j} \cdot \tau}
\]

where \( k_{A_j} \) and \( k_{B_j} \) are the first-order flotation rate constants for mineral A and B respectively and \( \tau \) is the average residence time of a particle in the flotation cell.

By rearranging Eq. (1), the relative rate constant (relative floatability, Gaudin (1957)) \( S_j \) can be expressed as a function of recovery of minerals A and B in cell \( j \) as follows:

\[
S_j = \frac{k_{A_j}}{k_{B_j}} = \frac{R_{A_j}}{1 - R_{A_j}} \cdot \frac{1 - R_{B_j}}{R_{B_j}}
\]

Cooper et al. (2004) found that relative floatability of sphalerite and pyrite was not dependent on the operational conditions \( (S_j \sim 2 - 3) \) and was approximately constant along the bank. Based on this, the
relative floatability is assumed to be constant for all cells in the bank, i.e., \( S_j = S, j = 1, \ldots, N \). Then, for a given recovery of mineral A in cell \( j \), the recovery of mineral B in that cell is given by:

\[
R_{Bj} = \frac{1}{1 + S \cdot (1 - R_{Aj}) / R_{Aj}}
\]  

(3)

Consider the flotation bank composed of \( N \) flotation cells depicted in Fig. 1. The optimization objective is: for a given target cumulative recovery of mineral A, find the cell-by-cell recovery profile of mineral A which maximizes the separation efficiency. This can be expressed mathematically as follows:

\[
\text{Max}_{R_{A1}, R_{A2}, \ldots, R_{AN}} E = \left( R_{A1}^c - R_{B1}^c \right)
\]  

(4)

subject to the following set of equality constraints:

\[
R_{A1}^c = R_{A1} + R_{A2} \cdot (1 - R_{A1}) + \cdots + R_{AN} \cdot (1 - R_{A1}) \cdots (1 - R_{AN-1})
\]

\[
R_{B1}^c = R_{B1} + R_{B2} \cdot (1 - R_{B1}) + \cdots + R_{BN} \cdot (1 - R_{B1}) \cdots (1 - R_{BN-1})
\]  

(5)

\[
S = \frac{R_{A1}}{1 - R_{A1}}, \quad j = 1, \ldots, N
\]

\[
R_{A1}^* = R_{\text{target}}
\]

and inequality constraints:

\[
0 \leq R_{Aj} \leq 1, \quad j = 1, \ldots, N
\]

\[
0 \leq R_{Bj} \leq 1, \quad j = 1, \ldots, N
\]  

(6)

where \( R_{A1}^c \) and \( R_{B1}^c \) are the cumulative recoveries of mineral A and B in the bank and \( R_{\text{target}} \) is the target cumulative recovery of mineral A.

To gain insight into the optimization problem, a bank composed of one, two and three cells is first analyzed. Then, the analysis is extended to the general problem consisting of a bank of \( N \) cells. The general optimization problem is formulated in the dynamic programming framework.

2.1. One-cell bank

This case does not entail any optimization problem since the end constraint \( R_{A1}^c = R_{\text{target}} \) completely determines the solution, i.e.:

\[
R_{A1}^* = R_{\text{target}}
\]  

(7)

where superscript * stands for optimal. Fig. 2 shows the separation efficiency of the cell as a function of recovery of mineral A for different relative floatability. There is a maximum in separation efficiency for a given relative floatability and that as separation becomes harder (decreasing relative floatability) the optimum operating point shifts towards lower recoveries. Since the end constraint imposed for the cumulative recovery in the cell fixes the recovery, the maximum separability is not attained at a target recovery (illustrated for \( R_{\text{target}} = 0.9 \)).
2.3. A three-cell bank

Fig. 5 shows the separation efficiency of the bank as a function of recovery of mineral A in the first two cells (which fixes the recovery in the third cell) for a target cumulative recovery of 0.75 (the target bank recovery for the Brunswick case).

Fig. 6 shows a representation of a flotation bank composed of N cells in the framework of dynamic programming. The next subsections describe the elements of the optimization problem, namely: states, decision variables, transformation equation, constraints and objective function.

2.4. General approach: Dynamic programming

The general problem is to optimize operation of a bank of N cells as in Fig. 1. The optimal operation is again defined as maximizing the separation efficiency for a given target cumulative recovery of mineral A. In this case, N degrees of freedom are available to solve the optimization problem, i.e., the recovery of mineral A in each cell. A brute-force approach is to quantize each cell recovery in discrete values and combine them to generate different cell-by-cell recovery profiles. The limitation of this approach is the so-called curse of dimensionality. For example, consider a bank of 7 cells and 20 discretized values for each cell recovery, then the number of possible recovery profiles rises to $20^7 = 1.280.000.000!$

An efficient method to deal with the optimization of serial structured processes is dynamic programming (Bellman, 1957). Applications in metallurgy and chemical engineering can be found in (Aris, 1964; Ray and Szekely, 1973; Maldonado et al., 2007).

2.4.1. State

The state variables are those variables that carry all the information about the operating condition of the stage. In our case, the selected state variables are the cumulative recoveries of mineral A and B in each cell:

$$\mathbf{x}_i = \begin{bmatrix} R_{A_i}^C \\ R_{B_i}^C \end{bmatrix}$$

(8)

where $R_{A_i}^C$ and $R_{B_i}^C$ are the cumulative recoveries of mineral A and B respectively up to cell $j$.

2.4.2. Decision

The decision or control variables are the available degrees of freedom to be manipulated in order to modify the operation of any stage. In our case these are the recovery of mineral A in each flotation cell, $R_{A_i}$:

$$\mathbf{u}_i = R_{A_i}$$

(9)

2.4.3. Transformation

The dynamic programming technique requires that the state variable of any stage depends only on the previous state and the decision variable of the actual stage. In our case, this is satisfied by the following transformation equation:

$$\mathbf{x}_i = \begin{bmatrix} R_{A_i}^C \\ R_{B_i}^C \end{bmatrix} = \begin{bmatrix} R_{A_{i-1}}^C + R_{A_i} \cdot (1 - R_{B_{i-1}}^C) \\ R_{B_{i-1}}^C + R_{B_i} \cdot (1 - R_{A_{i-1}}^C) \end{bmatrix}$$

(10)

with initial condition:

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(11)

2.4.4. Constraints

Constraints on a decision or state variable may appear from operational objectives, actuator constraints, etc. Four sets of constraints are considered in our optimization problem. The first one is an equality constraint imposing a target cumulative (bank)
recovery of mineral A; the second is that cell recoveries are bounded between 0 and 1; and the last set of constraints relates cell recoveries of mineral A and B with the relative floatability.

\[ R_{A0} = R_{\text{target}} \]
\[ 0 \leq R_A \leq 1, \quad j = 1, \ldots, N \]
\[ 0 \leq R_B \leq 1, \quad j = 1, \ldots, N \]
\[ S = \frac{R_A}{1 - R_A}, \quad j = 1, \ldots, N \]

(12)

2.4.5. Objective function

The optimization problem is formulated as maximizing the separation efficiency, determined by the difference between the cumulative recoveries of mineral A and B at the end of the bank subject to the transformation equations and constraints considered above. To this end, cell recoveries will be manipulated to yield an optimal recovery profile.

\[ \text{Max}_{R_{A0}, R_{B0}} E = \left( R_{A0} - R_{B0} \right) \]

As suggested from the previous optimization exercises with banks of 2 and 3 cells, the optimal solution obtained for N cells by applying the dynamic programming technique is a flat cell-by-cell recovery profile.

Considering the flat recovery profile, Fig. 7 (Top) shows that the optimal separation efficiency increases as the number of cells in the bank increases. Separation efficiency is sensitive to number of cells up to about 6 with reducing impact as more cells are added. Taking the 6-cell result we call attention to the fact that separation of cells up to about 6 with reducing impact as more cells are added. The analyses has shown that the way material is distributed down the bank (the recovery profile) influences separation between floatable (i.e., true-floating) minerals. For the case of constant relative floatability examined here the maximum separation at a target bank recovery is given by a flat recovery profile. There is a question how robust is the model (Eq. (11)) and how general the result.

The first-order fully mixed kinetic model is the basis of many, if not most, analyses of flotation systems (Weiss, 1985; Lynch et al., 1981a; Gupta and Yan, 2006). The model is applied both to the collection process in the pulp zone, where \( k = k_c \) (say), and to overall flotation for the fully mixed case where \( k = k_f \) with \( R_f \) being froth zone recovery (Finch and Dobby, 1990).

Frequently a constant rate constant is used (Lynch et al., 1981a; Sutherland, 1981; Jowett and Sutherland, 1985; Loveday and Broukaert, 1995) and interestingly this will yield the same conclusion found here for optimum bank operation. The assumption of constant relative floatability is potentially more defendable than assuming a constant \( k \) and avoiding having to specify \( k \) and work with time. Predicting the impact of other than constant relative floatability in principle is tractable. The model recognizes that the recoveries of two floatable minerals are linked; that if only physical variables are manipulated (like air rate) then a change in recovery of one mineral entails some proportional change in the second mineral, which Eq. (3) expresses.

While model choice and assumptions may attract debate, the notion of recovery profiling merits consideration. Fig. 7 introduces a practical point: maximum bank separation efficiency can exceed that of an individual cell provided the bank has about six cells. There are cost incentives to reduce the number of cells in a bank but there may be a price of reduced separability. In fact, the model points to the best operating point in terms of separation is to have a large number of cells (theoretically an infinite number) with each cell recovering a small increment (theoretically approaching zero). Fig. 8 explores this feature, showing recovery of mineral B versus recovery of mineral A in one cell for different relative floatabilities.

3. Discussion

It is evident that for any \( S > 1 \), the minimum sensitivity of \( R_B \) to \( R_A \) (i.e., least slope) is found when operating near the origin, i.e., \( R_B = R_A = 0 \). Fig. 8 also explains the asymmetries found in separation efficiency as a function of recovery (Figs. 2–5) since as recovery of mineral A increases beyond a certain point, ca. 0.7, the sensitivity of \( R_B \) to \( R_A \) significantly increases reducing separation.

Fig. 7. Top: Optimal separation efficiency as a function of number of cells in the bank for target cumulative recovery of mineral A of 0.9 and different relative floatability. Bottom: Optimal cell recovery of mineral A as a function of number of cells in the bank.

Fig. 8. Recovery of mineral B versus recovery of mineral A in a cell.
Much as Fig. 2 is a property of a cell, we can suggest Fig. 7 illustrates properties of a bank.

While not aware of recovery profiling being practiced we do know that the related concept mass recovery profiling is being explored for bank control (Supomo et al., 2008). The analysis here could be adapted to examine the impact of mass recovery profile on separation. To set either a mass or recovery profile translates into a local problem of selecting variables to control cell recovery. Air rate profile is perhaps the best local control. An increasing air rate profile does tend to distribute material down the bank by throttling recovery in the first cells which often "over-produce". This effect may contribute to the success of the increasing air profile.

The picture, however, is not complete until entrainment is included. It was evident at Brunswick that the increasing air profile had reduced entrainment by restricting recovery in the first couple of cells of the bank whereas the current analysis only hints that separation between the two floatable minerals sphalerite and pyrite was also enhanced. A bank property in our favour is that if to control entrainment the first cells are operated below optimum for maximum separation efficiency A from B this is less detrimental to separation efficiency than a strategy calling for increased recovery of A in the first cells. We continue to explore ways to include entrainment in the modelling effort. It may be possible to introduce a relative floatability of mineral to water which is independent of physical variables (Lynch et al., 1981b). An ambition is to determine the optimum recovery profile as set point to a froth vision system to control mass flow rate (recovery) from each cell in the bank.

4. Conclusions

Using a conventional first-order fully mixed model a flat cell-by-cell recovery profile has been found optimal in the sense of maximizing the separation efficiency of a bank for a given target cumulative recovery when the relative floatability is constant. The optimal solution is independent of the value of the relative floatability as long as it is constant down the bank. Air profiling is an efficient way to control recovery profile and may help explain the success of this strategy. Exploring the model it is found that separation efficiency increases with number of cells in the bank up to ca. 6 with diminishing gains above this number. A bank with six cells operating with the flat recovery profile will exceed the separation efficiency achievable in a single cell.

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