A new modification in Plitt’s equation for hydrocyclones simulation

A.C. Silva, E.M.S. Silva and J.D.V. Matos
INTRODUCTION

- Hydrocyclones major application is **granulometric classification** in closed circuit milling, but it is possible to highlight its operation in desliming, selective classification, thickening and pre-concentration.

- It is a **versatile, high capacity and without moving parts** equipment.
Within the empirical models used to hydrocyclones dimensioning the most used is the model proposed by Plitt (1976). This model can be used to predict the hydrocyclones operation without additional experimental data for a wide range of operational conditions.
HYDROCYCLONE EMPIRICAL MODELS

\[ d_{50c} = \frac{50.5 \, D_c^{0.46} \, D_i^{0.6} \, D_0^{1.21} \, e^{0.063 \, \phi}}{D_u^{0.71} \, h^{0.38} \, Q^{0.45} \, (\rho_s - \rho_l)^{0.5}} \]

Where:

- \( Q \) is the volumetric flow rate of hydrocyclone feed [l/min];
- \( \rho_l \) is the liquid phase density [g/cm³];
- \( \rho_s \) is the solid density [g/cm³] and
- \( \phi \) is the volumetric fraction of solids in the feed [%].
Addiction of the fluid dynamic viscosity ($\mu$)
Calibration factor ($k_1$) dependent on the solid fed in the hydrocyclone and a constant ($\alpha$) dependent on the feed flow rate.
High volumetric fraction of solids pulp and the difference between the solid and the liquid density is raised by one (not by 0.5)
\[ d_{50c} = \frac{50.5 D_c^{0.46} D_i^{0.6} D_o^{1.21} e^{0.063\phi}}{D_u^{0.71} h^{0.38} Q^{0.45} (\rho_s - \rho_l)^{0.5}} \]  
Plitt (1976)

\[ d_{50c} = k_2 \frac{2.6892 D_c^{0.46} D_i^{0.6} D_o^{1.21} \mu^{0.5} e^{0.063\phi}}{D_u^{0.71} h^{0.38} Q^{0.45} (\rho_s - \rho_l)^{0.5}} \]  
Gupta e Yan (2006)

Calibration factor \((k_2)\)
\[ d_{50c} = \frac{50.5 \, D_c^{0.46} \, D_i^{0.6} \, D_o^{1.21} \, e^{0.063 \phi}}{D_u^{0.71} \, h^{0.38} \, Q^{0.45} \left( \rho_s - \rho_l \right)^{0.5}} \]

Plitt (1976)

\[ d_{50c} = \frac{52.45 \, D_c^{0.46} \, D_i^{0.6} \, D_o^{1.21} \, e^{0.063 \phi}}{D_u^{0.71} \, h^{0.38} \, Q^{0.45} \left( \rho_s - \rho_l \right)^{0.5}} \]

Luz (2005)

Another calibration factor
HYDROCYCLONES TYPICAL GEOMETRIES

- Three main hydrocyclones families:

<table>
<thead>
<tr>
<th>Family</th>
<th>$D_i/D_c$</th>
<th>$D_o/D_c$</th>
<th>$I/D_c$</th>
<th>$L/D_c$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bradley</td>
<td>0.133</td>
<td>0.200</td>
<td>0.330</td>
<td>6.850</td>
<td>9.0°</td>
</tr>
<tr>
<td>Krebs</td>
<td>0.267</td>
<td>0.159</td>
<td>-</td>
<td>5.874</td>
<td>12.7°</td>
</tr>
<tr>
<td>Rietema</td>
<td>0.280</td>
<td>0.340</td>
<td>0.400</td>
<td>5.000</td>
<td>15 - 20.0°</td>
</tr>
</tbody>
</table>
MATHEMATICAL MODEL PROPOSITION

- Hydrocyclone operation simulations were realized to generate the proposed mathematical model using three hydrocyclone geometries.

- The adopted pulp was composed by iron ore ($\rho_s = 3.53 \text{ g/cm}^3$) and water ($\rho_l = 1.00 \text{ g/cm}^3$).
MATHEMATICAL MODEL PROPOSITION

- The volumetric fraction of solids in the feed \((\phi)\) assumed eleven different values: 0.5, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0, 45.0 and 50.0%.

- Ten volumetric flow rate \((Q)\) of hydrocyclone feed were used (from 0.5 to 5.0 m\(^3\)/h with increment equals to 0.5 m\(^3\)/h).
# MATHEMATICAL MODEL PROPOSITION

<table>
<thead>
<tr>
<th>Family</th>
<th>$D_c$ [cm]</th>
<th>$D_i$ [cm]</th>
<th>$D_o$ [cm]</th>
<th>$D_u$ [cm]</th>
<th>h [cm]</th>
<th>$\theta$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bradley</td>
<td>10.0</td>
<td>1.33</td>
<td>2.00</td>
<td>1.00</td>
<td>65.20</td>
<td>9.0</td>
</tr>
<tr>
<td>Krebs</td>
<td>10.0</td>
<td>2.67</td>
<td>1.59</td>
<td>1.00</td>
<td>54.74</td>
<td>12.7</td>
</tr>
<tr>
<td>Rietema</td>
<td>10.0</td>
<td>2.80</td>
<td>3.40</td>
<td>2.50</td>
<td>46.00</td>
<td>20.0</td>
</tr>
</tbody>
</table>
MATHEMATICAL MODEL PROPOSITION

- Altogether **330 simulations (110 simulations for each geometry)** were realized. Exponential regressions were made using the corrected cut size calculated in the simulations for each geometry, intending to recalculate Plitt’s model coefficients and therefore proposing a model which fits better to the calculated data.
Proposed model – constant model and the adoption of fluid dynamic viscosity in agreement with equation proposed by Plitt et al (1980)

\[ d_{50c} = \frac{50.5 \, D_c^{0.46} \, D_i^{0.6} \, D_o^{1.21} \, e^{0.063\phi}}{D_u^{0.71} \, h^{0.38} \, Q^{0.45} \left( \rho_s - \rho_l \right)^{0.5}} \]  

Plitt (1976)

\[ d_{50c} = \frac{2.54 \, D_c^{0.46} \, D_i^{0.6} \, D_o^{1.21} \, \mu^{0.5} \, e^{0.063\phi}}{D_u^{0.71} \, h^{0.38} \, Q^{0.45} \left( \rho_s - \rho_l \right)^{0.5}} \]
Proposed model (red line) versus simulated data (black dots) for a Rietema hydrocyclone operation with 4.5 m³/h of iron ore pulp.
PROPOSED MODEL VALIDATION

- Experimental data from 60 hydrocyclone tests published by Vieira (2006) were used to validate the proposal model.
- Phosphate ore ($\rho_s = 2.98$ g/cm$^3$) pulps in a wide range of volumetric fraction of solids and volumetric flow rate feed.
- 15 different hydrocyclone geometries were tested with 4 different volumetric flow rate of hydrocyclone feed.
Correlation between experimental and calculated $d_{50c}$ using Plitt’s model (1976) for all 60 hydrocyclone tests
Correlation between experimental and calculated $d_{50c}$ using Plitt's model (1976) for 40 hydrocyclone tests (purging the worst results)
Correlation between experimental and calculated $d_{50c}$ using the proposed model for all 60 hydrocyclone tests

Sample size 60
Correlation coefficient $r = 0.8820$ 
Significance level $P < 0.0001$ 95
% Confidence interval for $r$ 0.8093 to 0.9281
CONCLUSIONS

Through the study of the models based on Plitt’s model (1976) was possible to propose a new empirical model.

It is possible to affirm that the proposed model is more able of accurate calculate the $d_{50c}$ for pulps with low volumetric fraction of solids in the feed.
CONCLUSIONS

- More experimental work are needed to test the new model against wide chances in operational variables, pulps with two or more types of ores, fluid dynamic viscosity, volumetric flow rate of hydrocyclone feed and even pulp temperature.
ACKNOWLEDGES

The authors thank financial support from the Brazilian agencies CNPq, CAPES, FAPEG and FUNAPE.